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SciVerse ScienceDirect
Energy Procedia 16 (2012) 1401 – 1407
Energy Procedia

2012 International Conference on Future Energy, Environment, and Materials

Study on the Multi-products Dynamic Pricing Model under Uncertain Demands*

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Abstract

The multi-products dynamic pricing problem based on manufacturer was studied, in which the demand of purchasers was uncertain. The cost of the raw-material, production of multi-products, the stock and the cost of executing order from downstream enterprise were considered, the unsatisfied demand and the lead time in delivering were taken into account either. The time-space network was introduced to describe this dynamic supply chain network, by which the mathematical model for dynamic pricing that make manufacturers get the biggest profit was established. Finally, a numerical example was analyzed to prove the rationality of this model. The optimal price, output and stock from the numerical example can provide manufacturers with subsidiary decision-making to formulate reasonable production plan.

keywords :dynamic pricing; time-space network; uncertain demands; multi-products

1.I ntroduction

With the increasingly fierce market competition and the segmentations of customer group, more and more companies are seeking for dynamic pricing strategy. Especially, the appearance of network sales and the application of high technology, the research under uncertain demands are becoming more and more important. However, manufacturers in the upstream of supply chain can't usually determine effectively the demand of consumers and the market receptivity of price, so the price and quantity are decided by market and the utility of product was increased by a good stock management. If these can be

* This research is supported by the National Natural Science Foundation（10426011）and the Science and Technology Foundation of Heilongjiang Province for Youths (QC07C116).

 doi:10.1016/j.egypro.2012.01.222

estimated ahead of time, we can decide the quantity of production and reduce the stock, so that the nonessential expense can be avoided and the cost can be saved and the profit can be increased. Therefore, the research of dynamical pricing strategy is very important in reality.

Kincaid and Darling have first studied the single product dynamic pricing question, and constructed the run-on time stochastic dynamic programming model, as well as have given revenue function nature [1].Gallego and Ryzin study the aviation network market, which supposed the demand is stochastic and changes with price and it conforms to the Poisson's distribution. The research shows the decided price must be gather the obedience price set and changes along with the expectation profit [2].Zhao and Zheng study the expectation profit maximum with the dynamic pricing and the loss of unmet needs is not considered This research obtains: The most superior price can be enhanced along with the stock reduction in a period of time, simultaneously; the sales can be increased when the supply do not increase with the time but the ability limit of customer increases [3].Monahan etc establish a dynamic optimization model， which suppose that time is reverse with demands, finally he get the conclusion that the price and the expected profit of each period will affect each other[4]. Bertsimas and Boar pointed out that the effective multi-product pricing and inventory management can be applied to production planning [5]. Guo zhe etc studied dynamic pricing model for durable products in competitive environment of Electronic Commerce. Multi-period two stages pricing is adopted. Cost-plus model and customer-expectation model were selected for each stage respectively, and integrated for one pricing period [6]. Vitali Gintner study multi-station and multi-vehicle scheduling problem with time-space network and point out that the time-space network can completely express the class concept of the time and space in dealing with the dual problem of time and space [7].

Through analyze formerly literatures, the dynamic pricing researches few take the manufacturer as the subject, and they have majority entered researches and the analysis to retail merchant's transport business pattern, which have neglected the supply quantity and cost of raw material and order time difference relations of downstream buyer and manufacturer. Therefore this research will take manufacturer on supply chain as the object, and consider upstream raw material supplier and the downstream buyer group the supply chain net structure of this paper. While discuss relations of space and time, by the space and time network to express the question has a bigger help. Moreover, when discuss dynamic pricing question, the time factor must be considered, the price will change because of the time, the formerly dynamic pricing literatures little mentioned interactive relations of time and space, so this research will introduce the space-time network, and consider the time and space relation of the supply chain production and sealing process, then this paper will describe question and construct the mathematical model.. The cost of the raw-material, production, the stock and the cost of executing order from downstream enterprise were considered, the unsatisfied demand and the lead time in delivering were taken into account either. The time-space network was introduced to describe this dynamic supply chain network, by which the mathematical model for multi-products dynamic pricing that make manufacturers get the biggest profit was established. Finally, a numerical example was analyzed to prove the rationality of this model

2.Problem Descriptions

When $t=0$, the manufacturer firstly buys raw material from the upstream supplier and the raw material arrive, then the manufacturer to product, therefore the manufacturer can decide the expectation output. At the same time, the manufacturer will decide the price in the next period according to the demand of consumer and the downstream manufacturer make order based on this price and the order is unable to cancel, thus the manufacturer know the actual demand of downstream buyers.

When $t=1$, the manufacturer is wise to the stock or shortage according to the sales and output in this period. Considering the stock or shortage, the manufacturer forecasts the demands of next period, according to which to product. Having the same process with $t=0$, the buyer makes order based on the price at that time.

After $t=1$, the process of all periods will be followed by analogy.

3.The multi-products dynamic pricing model

The parameters of the model as follows:

M^j	The Set of all suppliers of raw materials
$c_{g,j}$	Unit raw material purchasing cost of g product
$c_{g,j}^T$	Unit raw material transportation cost of g product
c_g^F	The unit cost of production for the g product
c_g^o	The unit cost of services for the g product
c_g^s	The unit cost of stock for the g product
c_g^I	The unit shortage cost for the g product
y_g^t	The Sales quantity at t for the g product
R_g^t	The reality demands quantity at t for the g product
a	The slope item of demand function
b	The nodal increment item of demand function
$D_{g,j}^t$	Demands of g product for the j raw material at t
X_g^t	The output of g product at t
Q_g^t	The g product supply at t
P_g^t	The price of g product at t
S_g^t	The stock of g product at t
I_g^t	The shortage of g product at t
J^t	The expectation profit of the manufacturer at t
Ω^t	Sum cost of stock and shortage at t
Z_i^t	Value is 1 when having stock, otherwise is 0
$Normal(u,\sigma^2)$	Normal distribution that Mean value is u and Standard deviation is σ

3.1.Mathematic model:

The target functions of the first stage:

$$Max\ J^t = \sum_g (y_g^t \times P_g^t + I_g^{t-1} \times P_g^{t-1} - (c_{g,j} + c_{g,j}^T) \times D_{g,j}^t - c_g^F \times X_g^t - c_g^o \times Q_g^t) \tag{1}$$

Constraint condition of the first stage:

- The process of purchasing raw material:

$$D_{g,j}^t \geq X_g^t \qquad \forall j \in M^j \tag{2}$$

- The process of producing:

$$X_g^t + I_g^{t-1} - S_g^{t-1} = Q_g^t \tag{3}$$

- The demand function

$$P_g^t = aR_g^t + b \tag{4}$$

- The sales quantity:

$$y_g^t \ = \ \min \ (Q_g^t, R_g^t) \tag{5}$$

and $D_{g,j}^t \geq 0 \ \ \forall j \in M^j, X_g^t \geq 0, Q_g^t \geq 0, P_g^t \geq 0, \forall t$

The target functions of the second stage:

$$Min\ \Omega^t = \sum_g c^S \times s^t + c^I \times I^t \tag{6}$$

Constraint condition of the second stage:

The process of the supply will have stock or shortage.

$$R^t\left[Normal(u, \sigma^2)\right] - Q^t - I^t \leq M \times z_i^t \tag{7}$$

$$Q_g^t - R_g^t\left[Normal(u, \sigma^2)\right] - S_g^t \leq M \times (1 - z_i^t) \tag{8}$$

and $s_g^t, I_g^t \geq 0, z_i^t = 0 \text{or} 1$

3.2.The explains of the model

The model is divides into two stages for processing, type (1) is the target functions of the dynamic pricing for manufacturer to maximize expectation profit. First is the income, which is gained multiply price by the sales quantity and considering the stock or shortage quantity of the last period. If last has stock, the productivity of this period adds the stock of last period is the product supply of this period, that is to say the product can be stored; If last period has shortage, the productivity of this period subtracts the shortage of last period is the product supply of this period, that is to say the manufacturer considers the unmet needs and satisfies the shortage quantity of the last period firstly. This model takes the profit of manufacturer maximize as the goal. The income subtracts the cost, including the cost of raw material, the cost of production, the cost of services, namely for the obtained profit of manufacturer. Type(2) is the process expression that the manufacturer purchases raw material from the upstream raw material supplier, not only all the types of raw material that the product needs in form , but also the minimum quantity of all types of raw material is the productivity upper limit of manufacture. Type (3) is the process of production, which contains decision variables of productivity and supply and considers the stock or shortage of last period. Type (4) is the demand function, which the manufacturer in accordance with to formulate the price. Type (5) is the sales quantity, when the demand quantity of this period is bigger than the supply quantity namely does not have the stock, the sales quantity is the supply quantity; otherwise the sales quantity is the demand quantity, in other words, the sales quantity is the smaller in the supply quantity and demand quantity. The decision variables including the productivity, the supply quantity and the price are bigger than or equal to zero. The second stage is the process of cost penalty, whose goal is to cause the cost of stock and shortage to the smallest, as type (6) shows. The restraint can divide into two situations: one kind of situation is that the actual demand quantity of this period subtracts the product supply quantity of this period for the shortage quantity of this period, as type (7) shows; another kind of situation is that the product supply quantity of this period subtracts the actual demand quantity of this period for the stock quantity of this period, as type (8) shows. Only one kind of situation happens in the same stage, which is expressed by z_i^t.

The mathematical model is divides into two stages for processing. Firstly the optimum productivity, the product supply quantity and the price is decided in the first stage, then we lead the product supply quantity and the price to the second stage as parameters and extract the stock and shortage quantity, which brings back to the first stage as parameters, carries on repeatedly, until the optimal solution is found.

4.Numerical example studies

4.1.Descriptions of Numerical example

The upper limit capacity of the A is 2220 Tai and B is 2100 Tai. The unit total cost of raw material forAis3747.88 Yuan and for B is 2000.13Yuan, the unit cost of production

for A is 400Yuan and for Bis190Yuan, the unit cost of services for A is 114 Yuan and for B is1000Yuan, the unit total cost of the product for A is 4261.88 Yuan and for B is 3190.13 Yuan. The unit cost of stock is 3 percent of the unit total cost, the unit cost of shortage is half of the unit total cost. The demand curve is $p = -0.88R + 6357.3$ and the expense of the downstream buyers obey normal distribution that Mean value is1938 and Standard deviation is 110.2. This article will set about numerical example study with this example.

4.2.Mathematic model

The target functions of the first stage:

$$Max\ J^t = y_A^t \times P_A^t + y_B^t \times P_B^t + I_A^{t-1} \times P_A^{t-1} + I_B^{t-1} \times P_B^{t-1} - 3747.88 \times X_A^t$$
$$-2000.13 \times X_B^t - 400 \times X_A^t - 190 \times X_B^t - 114 \times Q_A^t - 1000 \times Q_B^t$$

Constraint condition of the first stage:

- The process of producing:

$$X_A^t + I_A^{t-1} - S_A^{t-1} = Q_A^t$$
$$X_B^t + I_B^{t-1} - S_B^{t-1} = Q_B^t$$

- The demand function :

$$P_g^t = -0.88R_g^t + 6357.3$$

- The sales quantity:

$$y_g^t = \min\ (Q_g^t, R_g^t)$$

- The capacity limit

$$X_A^t \le 2220$$
$$X_B^t \le 2100$$

and $X_g^t \ge 0, Q_g^t \ge 0, P_g^t \ge 0, \forall t$

The target functions of the second stage:

$$Min\,\Omega^t = 4261.88\times 0.03\times s_A^t + 3190.13\times 0.03\times s_B^t$$
$$+4261.88\times 0.5\times I_A^t + 3190.13\times 0.5\times I_B^t$$

Constraint condition of the second stage:

$$R_g^t\left[Normal(1938,110.2^2)\right] - Q_g^t - I_g^t \le 0$$
$$Q_g^t - R_g^t\left[Normal(1938,110.2^2)\right] - S_g^t \le 0$$

and $s_g^t, I_g^t \ge 0, z_i^t = 0 \text{or} 1$

4.3.Analysis of result

The software LINGO8.0 is used to compute. Five periods are taken for analysing the biggest profit of the manufacturer. The computation parameters and results of each period are showed in table1.

From table1, we can get:

- The demand of manufacturer is stochastic in each period; so the manufacturer adjusts the output of each period that the supply quantity changes with the stock or shortage of last period and the price of each period changes with the demand.
- Comparing the change trend of the stock/shortage to the demand in the 5 periods, we can know that the stock/shortage exist fluctuant state, but its extent is converging gradually.
- The demand quantity and price of product are inverse-correlation, which is connected with the demand function. The demand quantity is inverse ration with price in the demand function. The output and price also display inverse-correlation, whose cause is that the firm for getting more profit will try its best to satisfy the demand and the demand with price are inverse-correlation; therefore the output and price also become inverse-correlation.
- We can know from table1: The manufacturer obtains the smallest profit at the second and fifth period, whose reason is that they have shortage and their outputs have achieved the limit, The key of the manufacturer to enhance profit at this time is to improve the capacity; the manufacturer obtains the biggest profit at the third and first period, whose reason is that the supply is able to satisfy the demand, the key of the manufacturer to enhance profit at this time is to forecast the demand quantity accurately. The dynamic pricing model with uncertain demands not only sufficiently reflect the realistic condition and decide the output and price according to demand quantity of forecasting, but also cause the manufacturer to reduce cost and acquire higher profit.

TABLE I The Computation Result

period	demands		output		stock		price		profit
	A	B	A	B	A	B	A	B	
1	2155	2050	2155	2050	0	0	4460.9	4553.3	3223387
2	2300	2120	2220	2100	-80	-20	4333.3	4491.7	2879707
3	1963	2010	2043	2030	0	0	4629.86	4588.5	3593933
4	2210	2080	2210	2080	0	0	4412.5	4526.9	3113352
5	2230	2170	2220	2100	-10	-70	4394.9	4447.7	2928223

5.Conclutions

The time-space network architecture for multi-products dynamic pricing in the supply chain is established and it can fully reflect the process of the dynamic pricing with uncertain demand, which is proved by the numerical example; Considered the flow of production and the sales way of manufacturers, the mathematical model for dynamic pricing is established with time-space network; using software

LINGO8.0 for computing, the sampling-random demand is taken to the simulation test. After validation, the mathematical model is helpful for enterprise to maximize the expectation profit and minimize the cost of the stock and shortage.

This research only considers one customer; the following research can construct the dynamic pricing model with many customers, which more correspond to the reality operating condition in nowadays supply chain.

Acknowledgment

The authors would like to dedicate this paper to our teachers whose knowledge, enthusiasm for research, energy, guidance and friendship have been a true inspiration. We also would like to thank our families for their cares, understanding and the great support.

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